USN | | | | | | | | | | | 14MAR/MAU/IAE/MDE/MMD/MST/MTH/MTP11

# First Semester M.Tech. Degree Examination, Dec.2016/Jan.2017
# Applied Mathematics

Time: 3 hrs. Max. Marks:100

**Note:** ***Answer any FIVE full questions.***



1 a. Define with suitable examples: i) Significant figure; ii) Round off error; iii) Truncation error; iv) Inherent error; v) Relative error. **(10 Marks)**

b. The differential equation governing velocity of a falling parachutist is given by $\frac{dv}{dt} = g - \left(\frac{c}{m}\right)v$, where g-acceleration due to gravity, m-mass of parachutist, c-drag coefficient, obtain analytical solution at t = 10 sec. Also, obtain finite difference approximation in steps of 2 sec till t = 10sec, assuming v(0) = 0. Data: g = 9.8, m = 68.1, c = 12.5. **(10 Marks)**

2 a. Write important steps of bisection method to find the root of the equation f(x) = 0. Also, find a real root of the equation xsinx + cosx = 0 by using Newton-Raphson method, where $x_0 = 3\pi/4$. **(10 Marks)**

b. Apply Muller's method to find the root of the equation $x^3 - 3x - 5 = 0$ which lies in [2, 3]. **(10 Marks)**

3 a. Find all the roots of $x^3 - 4x^2 + 5x - 2 = 0$ by Graffe's root squaring method. **(10 Marks)**

b. Perform two iterations of the Bairstow method to extract a quadratic factor $x^2 + px + q$ from the polynomial $x^3 + x^2 - x + 2 = 0$. Use the initial approximations $P_0 = -0.9$, $q_0 = 0.9$. **(10 Marks)**

4 a. Find the minimum value of y from the table:

| x: | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|
| y: | 0.205 | 0.240 | 0.259 | 0.262 | 0.250 | 0.224 |

**(10 Marks)**

b. Use Romberg's method to compute $\int_0^1 \frac{dx}{1+x^2}$, correct to 4 decimal places, by taking h = 0.5, 0.25 and 0.125. **(10 Marks)**

5 a. Solve the system of linear equations $x_1 + 2x_2 + 3x_3 = 5$; $2x_1 + 8x_2 + 22x_3 = 6$ and $3x_1 + 22x_2 + 82x_3 = -10$ by using Cholesky method. **(10 Marks)**

b. Using partition method, find the inverse of the matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$$

**(10 Marks)**

6 a. Use Jacobi's method to find all the eigen values and the eigen vectors of the matrix $\begin{bmatrix} 2 & \sqrt{2} & 4 \\ \sqrt{2} & 6 & \sqrt{2} \\ 4 & \sqrt{2} & 2 \end{bmatrix}$ **(10 Marks)**

b. Transform the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ to tridiagonal form by given method. Use exact arithmetic. **(10 Marks)**

7 a. Let $A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}$, $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$ and define a transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ by $T(X) = AX$, so that $T(X) = AX = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 3x_2 \\ 3x_1 + 5x_2 \\ -x_1 + 7x_2 \end{bmatrix}$.

i) Find T(u), the image of u under the transformation T.

ii) Find an X in $\mathbb{R}^2$ whose image under T is b. **(06 Marks)**

b. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Then T is one-to-one if and only if the equation T(X) = 0 has only the trivial solution. **(06 Marks)**

c. In a certain region, about 5% of the city's population moves to the surrounding suburbs each year, and about 4% of the suburban population moves into the city. In 2000, there were 600,000 residents in the city and 400,000 in the suburbs. Set up a difference equation that describes this situation, where $x_0$ is the initial population in 2000. Then estimate the population in the city and in the suburbs two years later, in 2002. **(08 Marks)**

8 a. Let $y = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$ and $u = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$. Find the orthogonal projection of y onto u. Then write y as the sum of two orthogonal vectors, one in span {u} and one orthogonal to u. **(06 Marks)**

b. Show that $\{v_1, v_2, v_3\}$ is an orthonormal basis of $\mathbb{R}^3$, where $v_1 = \begin{bmatrix} 3/\sqrt{11} \\ 1/\sqrt{11} \\ 1/\sqrt{11} \end{bmatrix}$, $v_2 = \begin{bmatrix} -1\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}$, $v_3 = \begin{bmatrix} -1/\sqrt{66} \\ -4/\sqrt{66} \\ 7/\sqrt{66} \end{bmatrix}$. **(04 Marks)**

c. Find a least-squares solution of Ax = b for

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} -3 \\ -1 \\ 0 \\ 2 \\ 5 \\ 1 \end{bmatrix}.$$ **(10 Marks)**

* * * * *